Functional Analysis HW 5

1. Let Y be a closed subspace of a normed space X. Let $i : Y \to X$ be the natural inclusion and $\pi : X \to X/Y$ the natural projection. Show that

   (i) the adjoint operator $i^{**} : Y^{**} \to X^{**}$ is an isometry.

   (ii) the adjoint operator $\pi^* : (X/Y)^* \to X^*$ is an isometry.

   Consequently, Y^{**} and $(X/Y)^*$ can be viewed as the closed subspaces of X^{**} and X^* respectively.